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Localized structures in cellular flows

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Abstract. We consider Ginzburg–Landau-type models for localized structures observed in the vicinity of subcritical bifurcations to cellular flows, where two metastable homogeneous states coexist in an interval range of the control parameter. A localized structure consists of a small region in the bifurcated state surrounded by the basic state. We show how non-variational effects, i.e. the absence of a free energy to minimize, can explain the stability of these stuctures, contrary to the case of droplets in first-order phase transitions.

1. Introduction

Localized structures are widely observed in fluid flows. Well known examples are the local regions of turbulent motion surrounded by laminar flow, which develop in many open-flow experiments (e.g. pipe flow, channel flow, boundary layers) [1]. More recently, spatially localized standing surface waves have been observed on a horizontal layer of fluid submitted to vertical vibrations [2], and convection in binary fluid mixtures displayed localized travelling waves [3–6]. In all cases the possible origin of localized structures lies in the existence of a subcritical instability, which implies that two different homogeneous stable states coexist in an interval range of the control parameter. The simplest spatial non-uniformity consists of an interface between the two stable states. A similar situation occurs in first-order phase transitions, for instance when droplets of liquid nucleate in a supersaturated vapour. In phase transitions the droplets are always unstable; they either shrink or expand. In the instability problem, a 'droplet' consists of a region where the system is in the bifurcated state, surrounded by the basic state. A puzzling feature of localized structures in fluid flows consists of their stability on a finite interval range of the control parameter. We have proposed that non-variational effects, i.e. due to the non-existence of a 'free energy' to minimize (a Lyapunov functional), can stabilize the droplet-like structure [7]. We describe this mechanism in section 3 after a short presentation of the Ginzburg–Landau-type model (section 2).

2. Ginzburg–Landau-type amplitude equations

We consider a wave instability of a basic laminar state in a spatially homogeneous autonomous system; the order parameter of the bifurcated wavy regime is the complex amplitude $A(x,t)$ of the wavepacket of the form, $A(x,t)\exp[\mathrm{i}(\omega_0 t - k_0 x)]$ +

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$\overline{A}(x,t)\exp[-\mathrm{i}(\omega_0 t - k_0 x)]$. At the instability onset, the real part of the growth rate of A vanishes; thus it is assumed that the long-wavelength dynamics can be described asymptotically by expanding $\partial A/\partial t$ in Taylor series of A, $\partial A/\partial x$, $\partial^2 A/\partial x^2, \ldots$. As for the free energy in the vicinity of a phase transition, the form of this amplitude equation is determined by symmetry constraints, translational invariance in time and space, which implies that the amplitude equation should be invariant under rotations in the complex plane, $A \rightarrow A\exp(\mathrm{i}\theta)$. Thus, the amplitude equation is of the Ginzburg–Landau type

$$\frac{\partial A}{\partial t} = \mu\, A - c\,\frac{\partial A}{\partial x} + \alpha\,\frac{\partial^2 A}{\partial x^2} + |A|^2 A\; P(|A|^2). \tag{1}$$

The Fourier transform of the linear part of (1) corresponds to the complex growth rate, $\eta(k) = \sigma(k) + \mathrm{i}(\omega(k) - \omega_0)$, where $\sigma(k) = \mu - \alpha_r(k-k_0)^2 + \ldots$ is the growth rate and $\omega(k) = \omega_0 + c(k-k_0) - \alpha_i(k-k_0)^2 + \ldots$ is the dispersion relation. We assume $\alpha_r > 0$, thus small perturbations with a wavenumber k_0 are amplified first when $\mu > 0$. As the distance from criticality μ increases, there exists a band of linearly unstable wavenumbers of order $\sqrt{\mu}$. c is the group velocity and a non-zero value of α_i corresponds to dispersion. P is a polynomial in $|A|^2$; the sign of the real part β_r of its constant term β determines the super or subcritical nature of the instability, i.e. the order of the transition. When $\beta_r > 0$, small perturbations are not stabilized by the leading-order non-linearity, and the bifurcation is subcritical. This is the case of the experimental situations quoted above, and an equation analogous to (1) was derived long ago for the amplitude of two-dimensional Tollmien–Schlichting waves in the plane Poiseuille flow [8]. The imaginary part β_i describes an amlitude-dependent frequency of the wave, as usual for a non-linear oscillator.

The simplest way to prevent the instability blow-up in the subcritical case, is to consider a quintic non-linearity with a coefficient γ the real part of which is $\gamma_r < 0$. Thus, in the reference frame moving at the group velocity, the Ginzburg–Landau model is

$$\frac{\partial A}{\partial t} = \mu\, A + \alpha\,\frac{\partial^2 A}{\partial x^2} + \beta\,|A|^2 A + \gamma\,|A|^4 A. \tag{2}$$

Two limit cases are of interest, the conservative one and the variational one. In the dissipationless limit, the system is conservative and has time reversal symmetry, thus (2) should be invariant under $t \rightarrow -t$, $x \rightarrow -x$, $A \rightarrow \overline{A}$. Therefore the coefficients should be all pure imaginary, and one gets a modified non-linear Shrödinger equation. This equation has stable pulse-like [9] and unstable hole-like [10] soliton solutions. However, these solutions cannot explain the localized structures observed experimentally in dissipative systems far from equilibrium. The other limit is the variational one, obtained if the wave frequency does not depend on its amplitude. This is obviously true in particular for a stationary instability, $\omega = 0$. The space reflection symmetry implies that (2) should be invariant under, $x \rightarrow -x$, $A \rightarrow \overline{A}$. Thus, the coefficients of (2) should be all real, and (2) has a Lyapunov functional, $\mathcal{L}\{A\}$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{L}\{A\} = -\int_0^L \left|\frac{\partial A}{\partial t}\right|^2 \mathrm{d}x \leq 0 \tag{3}$$

where

$$\mathcal{L}\{A\} = \int_0^L \left(\frac{1}{2}\left|\frac{\partial A}{\partial x}\right|^2 - V(|A|)\right) \mathrm{d}x \tag{4}$$

with

$$V(|A|) = \tfrac{1}{2}\mu|A|^2 + \tfrac{1}{4}\beta_r|A|^4 + \tfrac{1}{6}\gamma_r|A|^6. \quad (5)$$

$\mathcal{L}$ decreases in time and is minimum for uniform solutions that maximize $V(|A|)$. There exists a particular value $\mu_p = 3\beta_r^2/16\gamma_r$ of the control parameter μ, for which the $A = 0$ and $A \neq 0$ uniform solutions have the same 'energy' $-V$. For $\mu = \mu_\mathrm{p}$, an isolated interface between the two uniform states remains at rest; this corresponds to the Maxwell plateau in first order phase transitions [11]†. Pulse-like solutions are always unstable; they either shrink or expand in such a way that the lowest 'energy' state increases in size. The 'variational pulse' is even unstable for $\mu = \mu_\mathrm{p}$ because of the interaction between its limiting interfaces. Therefore, the stability of pulse-like solutions can be explained only with a non-variational effect.

3. Stable pulse generated by a subcritical oscillatory instability

We have numerically integrated equation (2) with a pseudo-spectral method involving 512 complex modes and periodic boundary conditions on the interval $[0, L]$. For a variety of initial conditions and large interval range of the coefficients, α, β, we have observed stable pulse-like solutions as shown in figure 1 [7]. The pulses exist for values of μ within a finite band. It is important to note that their size does not depend on the box length. Notice that the amplitude of the pulse is strongly localized while its phase varies almost linearly in space.

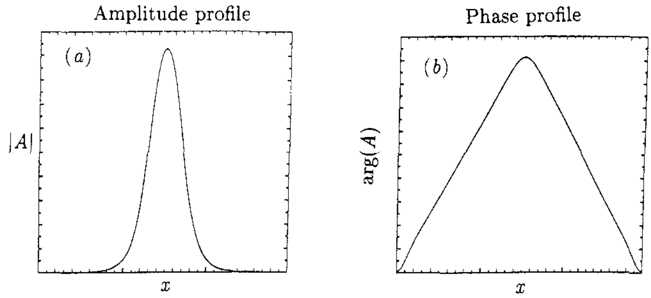

Figure 1. 1D pulse-like solution in the case $\alpha_i = 0$. Other parameters are $\mu = -0.1$, $\alpha_r = 1$, $\beta = 3 + \mathrm{i}$, $\gamma = -2.75 + \mathrm{i}$, interval length: $L = 30$. (*a*) Amplitude profile $|A(x)|$. (*b*) Phase profile $\theta(x) = \arg A(x)$.

Let us consider for simplicity equation (2) with $\alpha_i = 0$ that corresponds to figure 1, and try to understand the stability mechanism. The pulse-like solutions are of the form $A(x,t) = R_0(x)\exp\{\mathrm{i}[\Omega t + \theta_0(x)]\}$. In the outer region the pulse amplitude is very small and we can neglect non-linear terms. It is then easy to show that $R_0(x)$ decays exponentially, and that the phase gradient modulus is a constant determined by Ω. The travelling waves frequency correction, Ω, depends on the pulse shape $R_0(x)$ [7]. Writing the equations for $R(x,t)$ and $\phi(x,t)$, where $A(x,t) = R(x)\exp(\mathrm{i}\phi(x))$, $(\alpha = 1)$

† This argument is well known in phase transition theory. It was also applied for subcritical instabilities.

$$\frac{\partial R}{\partial t} = \left[\mu - \left(\frac{\partial \phi}{\partial x}\right)^2\right] R + \beta_r\, R^3 + \gamma_r\, R^5 + \frac{\partial^2 R}{\partial x^2} \tag{6a}$$

$$R\frac{\partial \phi}{\partial t} = \beta_i\, R^3 + \gamma_i\, R^5 + 2\left(\frac{\partial R}{\partial x}\right)\left(\frac{\partial \phi}{\partial x}\right) + R\,\frac{\partial^2 \phi}{\partial x^2} \tag{6b}$$

we notice that a linear variation in space of the phase ϕ simply renormalizes μ in the equation for R. As said above, we have for pulse-like solutions $\phi = \Omega\, t + \theta_o(x)$, where $(\partial\theta_0/\partial x)^2$ is constant in the outer region and vanishes in the pulse core. We define $\mu_{\text{eff}} = \mu - (\partial\theta_0/\partial x)^2$. The effect of the phase gradient is thus to decrease μ_{eff} in the outer region, maintaining it below the Maxwell plateau corresponding to the variational problem (6a) with $(\partial\phi/\partial x)^2$ constant. On the contrary, the pulse core μ_{eff} is above the Maxwell plateau if μ is large enough. The pulse gets stable by changing its shape $R_0(x)$, until its effect on Ω generates the correct value of the phase gradient and thus of μ_{eff}. This mechanism operates on a finite range of μ in the bistability region. It also works for two-dimensional fields $A(x, y, t)$ [7].

These pulses can be obtained perturbatively in the conservative and variational limits [12–16]. In the conservative limit, the effect of the slightly dissipative terms is to select the pulse size among a family of scale invariant solitons. One can also obtain dissipative analogues of the 'hole solitons' of [10].

Let us note finally that in many situations one must consider both the right- and left-travelling waves, $A_-\exp[\mathrm{i}(\omega t - kx)]$ and $A_+\exp[\mathrm{i}(\omega t + kx)]$ generated at the Hopf bifurcation. The coupled amplitude equations for A_- and A_+ have localized solutions, in particular the localized standing waves generated by a parametric excitation [2]. One can also study the dynamics during the collision of counter-propagating pulses [17, 18].

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